SHORTER ARTICLES AND DISCUSSION

THE TABULATION OF FACTORIAL VALUES[1]

In *Science* for January 23[2] Dr. Ellis L. Michael discusses the validity of the ordinary system of tabulation in the determination of the probable number of bacteria in an emulsion. He argues in favor of the use of the logarithms of the measurements instead of the direct measurements because the former give a symmetrical distribution, while the latter give one that is distinctly asymmetrical. As Dr. Michael has invited discussion it may be of interest to mention briefly a similar method used during the last two years in a study of the germinal and environmental factors affecting eye facet number in the bar races of *Drosophila.* A report of the method was made at the St. Louis meeting of the American Society of Zoologists and the results of its application to the particular problems in hand are being published in a series of papers.[3]

In working up the data it became evident that the demands of the biological analysis were not adequately met by the system of arrangement in classes with equal facet numbers. The wide range in individual stocks and the still wider differences between different races made it desirable to express relations directly in terms of factorial units affecting facet number rather than in facet numbers. In dealing with a stock averaging 30 facets as compared with one averaging 300 facets it became evident that a one facet change at the mean in a 30 facet stock represents the same factorial value as a ten facet change at the mean in a 300 facet stock and that *a corresponding principle applies within the range of a single stock.* Accordingly the classes were arranged

1 Contribution from the Zoological Laboratory of the University of Illinois, No. 152.

2 "Concerning Application of the Probable Error in Cases of Extremely Asymmetrical Frequency Curves," *Science,* N. S., 51: 89–91.

3 "A Change in the Bar Gene of *Drosophila* Involving Further Decrease in Facet Number and Increase in Dominance," *J. Gen. Physiol.,* 1919, 2: 69–71. *J. Exp. Zool.,* 1920, 30: 293–324.

so that the facet range of each class is a fixed per cent. of the mean facet value of *its class.* In other words the class facet ranges vary in such a way as to give the same logarithmic range to each class.

As an illustration eye facet counts in 488 females of the unselected white bar stock may be taken. The following table gives the frequency distribution obtained when the classes have the same facet ranges:

Facet Counts	Frequency in Per Cents.
16– 21	0.2
22– 27	0.2
28– 33	2.9
34– 39	10.9
40– 45	14.3
46– 51	12.3
52– 57	12.9
58– 63	11.7
64– 69	9.2
70– 75	8.0
76– 81	3.9
82– 87	3.9
88– 93	3.3
94– 99	2.9
100–105	1.4
106–111	1.6
112–117	0.2
118–123	0.0
124–129	0.0
130–135	0.2

The same arrangement is shown in graphic form in the following figure:

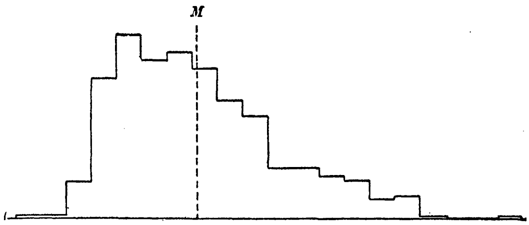

FIG. 1.

There is a marked positive skewness. The next table uses the same original data but with the facet range in any class equal to ten per cent of the mean of *that* class:

Facet Counts	Frequencies in Per Cents.	Factorial Units from Mean
20– 21	0.2	— 9.93
22– 23	0.0	— 8.93
24– 26	0.2	— 7.93
27– 29	0.6	— 6.93
30– 32	1.8	— 5.93
33– 35	3.1	— 4.93
36– 39	8.2	— 3.93
40– 43	9.6	— 2.93
44– 48	11.1	— 1.93
49– 53	11.9	— 0.93
54– 59	11.1	+ 0.07
60– 65	11.3	+ 1.07
66– 72	10.4	+ 2.07
73– 80	5.9	+ 3.07
81– 88	5.1	+ 4.07
89– 97	5.5	+ 5.07
98–107	2.3	+ 6.07
108–118	1.4	+ 7.07
119–131	0.0	+ 8.07
132–145	0.2	+ 9.07

The following figure is based on the same arrangement:

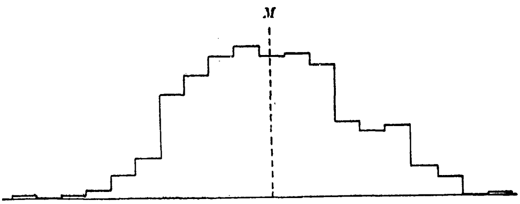

FIG. 2.

This is much closer to a normal distribution of frequencies than in the ordinary method. It is correspondingly more reliable in the determination of the various constants.

If the biological assumption upon which this tabulation is based is correct the classes are of equal value as far as the *factors* affecting facet numbers are concerned though the facet ranges are different. In following out this view the intervals on the variation scale have been expressed in terms of class units, each unit being equivalent to a factor which produces a change of ten per cent. in facet number. Some arbitrary point, for instance the mean of the unselected stock, may be taken as the point of reference or zero and every facet value has a corresponding factorial value on the scale. The variation constants may be ob-

tained in the ordinary way but in terms of factorial units and not facet units. The standard deviations are used directly as coefficients of variation.

The biological validity of the factorial method as given is of course dependent upon the correctness of the view that eye facet numbers have such a relation to environmental and germinal factors as is indicated. The normality of the factorial distribution has already been mentioned. General embryological considerations favor proportionate action of factors rather than action by accretion. But I wish to mention particularly the definite experimental proof that at least one factor, temperature, is in agreement with the hypothesis. Seyster[4] has shown that in bar eye facet number decreases with increase in the temperature at which the larvæ of *Drosophila* are reared. This decrease follows van't Hoff's law if an inhibitor of facet number is assumed as the effective agent upon which the temperature acts. Krafka[5] has demonstrated that this general law applies to ultra-bar as well as to bar eye and that for the different bar stocks the effect of a degree of change in temperature is roughly proportional to the mean value of the stock and the same is approximately true for the effects of a degree of change in temperature throughout the range of a single stock. The following table gives the facet values for ultra-bar, low selected bar and unselected bar at 15° and 25°:

15° Facet Values	25° Facet Values	Differences	Ratios of Differences
51.5	25.2	26.3	1.0
189.0	74.2	114.8	4.4
269.8	120.5	149.3	5.7

Representing the effect of a ten-degree difference for ultra-bar as unity, low selected bar has 4.4 times and unselected bar 5.7 times this difference. It is obvious that difference in facet number is not a good measure of the value of the temperature factor.

On the other hand, if facet values are reduced to factorial values according to the method given above the results are as follows:

4 Seyster, E. W., "Eye Facet Number as Influenced by Temperature in the Bar-eyed Mutant of *Drosophila melanogaster* (*ampelophila*)," *Biol. Bull.*, 1919, 37: 168–182.

5 Krafka, Joseph, Jr., "The Effect of Temperature upon Facet Number in the Bar-eyed Mutant of *Drosophila*," *J. Gen. Physiol.*, 1920. (In press.)

15° Factorial Values	25° Factorial Values	Differences	Ratios of Differences
− 0.83	−7.86	7.03	1.0
+12.17	+2.79	9.38	1.3
+15.72	+7.73	7.99	1.1

This is a much closer approach to unity for the ratios than in the case of facet values and the units employed may be taken as fairly close measures of the temperature factor.

A change of one facet is therefore not of equal factorial value at different points on the variation scale as far as temperature is concerned. A plotting of the data using facets as the units does not give a uniform factorial scale. Suppose temperature to be the only factor causing variation in the facet number of a particular stock but knowledge of the actual temperatures involved in the production of a particular population to be lacking and it is desired to derive the value of the temperatures from the facet values. Obviously the closer approximation is obtained by the tabulation in which each class has a facet range equal to a definite per cent. of its facet mean. Krafka's data show that even in this case the determination is not exact but certainly the error is of a much lower order than that involved in using facets as the units.

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AN EXPERIMENT ON REGULATION IN PLANTS[1]

IT is a fundamental fact that of the enormous number of buds on a tree only a few of these normally develop into branches. Every bud, however, has the capability of growth and will grow into a branch if the more apical bud or buds are removed. Even normally, in uninjured trees, some of the lateral buds grow into

[1] After this paper was written, my attention was called to an article by Child and Bellamy (*Science*, N. S., L, 362, 1919), in which somewhat similar experiments were reported and the same conclusion arrived at. Physiological isolation of two regions of a whole plant was produced by low temperature instead of by actual killing of tissue as in my experiments. In view of the importance of growth phenomena I believe it worth while to again call attention to the conclusions to be drawn from these facts, especially as the experiments of Child and Bellamy refer only to the influence of a growing stem on the growth of other stems and not to the influence of growing roots on the development of roots in other regions of a plant.